IMPERIAL COLLEGE LONDON

Design Engineering MEng EXAMINATIONS 2021

For Internal Students of the Imperial College of Science, Technology and Medicine
This paper is also taken for the relevant examination for the Associateship or Diploma

DESE50002 – Electronics 2

SOLUTIONS

Date: 28 April 2021 10.00 to 11.30 (one hour thirty minutes)

This paper contains 6 questions.
Attempt ALL questions.

The numbers of marks shown by each question are for your guidance only; they indicate approximately how the examiners intend to distribute the marks for this paper.

This is an OPEN BOOK Examination.

1. a) Show that the signal shown in *Figure Q1* can be modelled mathematically by the following equation. $u(t)$ is the unit step function.

$$x(t) = (t-1)u(t-1) - (t-2)u(t-2) - u(t-4)$$

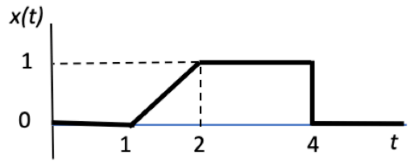


Figure Q1

[4]

b) Given that $s = \sigma + j\omega$ is the complex frequency, show that

$y(t) = \frac{1}{2}\left(e^{st} + e^{s^*t}\right) = e^{\sigma t}\cos\omega t$ where s^* is the conjugate of s.

[2]

Sketch $y(t)$ for the cases where $\sigma < 0,\ \sigma = 0\ and\ \sigma > 0.$

[6]

c) Based on the definition of the impulse function $\delta(t)$, show that the following equations are correct.

(i) $(t^3+3)\delta(t) = 3\delta(t)$

[2]

(ii) $\frac{\omega^2+1}{\omega^2+9}\delta(\omega-1) = \frac{1}{5}\delta(\omega-1)$

[3]

This question tests students' understanding of signal modelling, some basic mathematically representation of signals and the shift theorem.

a) t u(t) is a linearly ramp from 0 with gradient of 1. Therefore the (t-1)u(t-1) is the ramp signal shifted to t = 1.
(t – 2) u(t – 2) is the same ramp but delay but t = 2. The subtraction cancels the first term and result in the flat part between t = 2 and 4. Finally, -u(t-4) is a negative unity step function delayed by 4, which brings the signal back to zero.

b)

$$\begin{aligned} y(t) &= \frac{1}{2}\left(e^{st} + e^{s^*t}\right) \\ &= \frac{1}{2}\left(e^{(\sigma+j\omega)t} + e^{(\sigma-j\omega)t}\right) \\ &= e^{\sigma t}\frac{1}{2}\left(e^{j\omega t} + e^{-j\omega t}\right) \\ &= e^{\sigma t}\cos\omega t \end{aligned}$$

c) (i) $(t^3+3)\delta(t) = t^3\delta(t) + 3\delta(t)$

$\delta(t) = 1$ only when t = 0, and is 0 otherwise.

Therefore the first term must be 0 and only the 2nd term remains.

(ii) $\delta(\omega-1)$ is 1 only when $\omega = 1$.

Therefore, we substitute $\omega = 1$ to $\frac{\omega^2+1}{\omega^2+9}$, and this gives: $\frac{1}{5}\delta(\omega-1)$.

2. The equation below describes the signal $x(t)$.

$$x(t) = 0.5\sin(1000\pi)\,t + \delta(t) + 1.5\,u(t)$$

a) Sketch the waveform of the signal $x(t)$ for -5ms ≤ t ≤ 5ms.

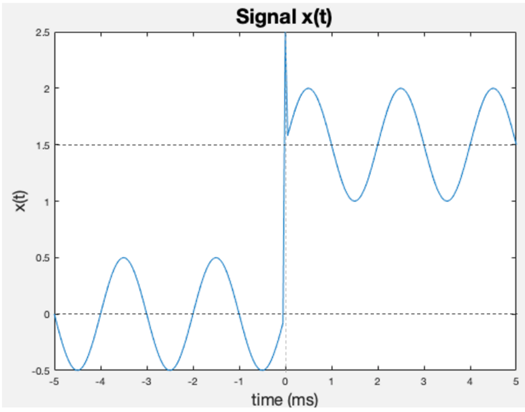
[6]

b) By referring to the Fourier Transform table, sketch the absolute amplitude spectrum $|X(\omega)|$ as a two-sided spectrum (i.e. with both positive and negative frequency ω on the x-axis).

[7]

This question tests students on their ability to translate a signal equation to a signal waveforms, to show how a complex time domain signal can be decomposed to separate signals, and how it is mapped to the frequency domain.

a) The signal consists of three components are: a sinewave with a period of 2msec (fs=500Hz) and zero phase, with amplitude of 0.5, a Dirac function and a unit step function with step of 1.5.

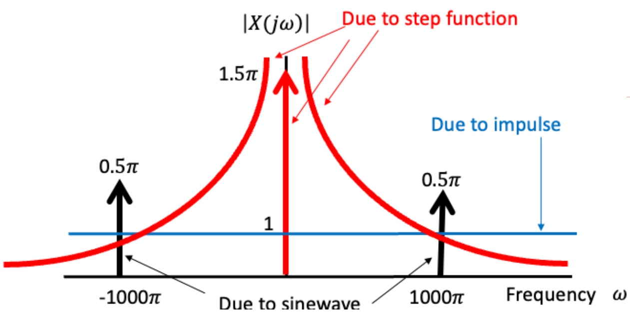


b) Since Fourier Transform is linear, we can combine the spectrum of each signal component to obtain the spectrum of x(t). We are only interested in the absolute amplitude.

From Lecture 3 slide 14 & 15, we have the following:

6	$\delta(t)$	1
10	$\sin\,\omega_0 t$	$j\pi[\delta(\omega+\omega_0)-\delta(\omega-\omega_0)]$
11	$u(t)$	$\pi\delta(\omega)+\dfrac{1}{j\omega}$

Therefore, the spectrum $|X(\omega)|$ is 500Hz is the same as $\omega = 1000\,\pi$ rad/sec.



3. a) When and why does aliasing happen in a sampled data system? What are the bad consequences of aliasing? How can this be avoided?

[6]

b) A musical chord consists of three notes with identical amplitude A: E4 at 330Hz, G4 at 392Hz and C5 at 523Hz.

(i) The chord signal y(t) is sampled at a rate of 8kHz. Sketch on your paper sheet the one-sided spectrum |Y(f)| of the sampled signal over the frequency range of 0Hz to 10kHz.

[10]

(ii) If the signal is sampled at 1kHz instead, what is the frequency of the aliased component?

[4]

This question tests students' understanding of sampling, sampling theorem, aliasing and frequency folding. The three notes in the cord is taken from the Ring Door Bell chime – not relevant to the question, but is where I got the idea from!

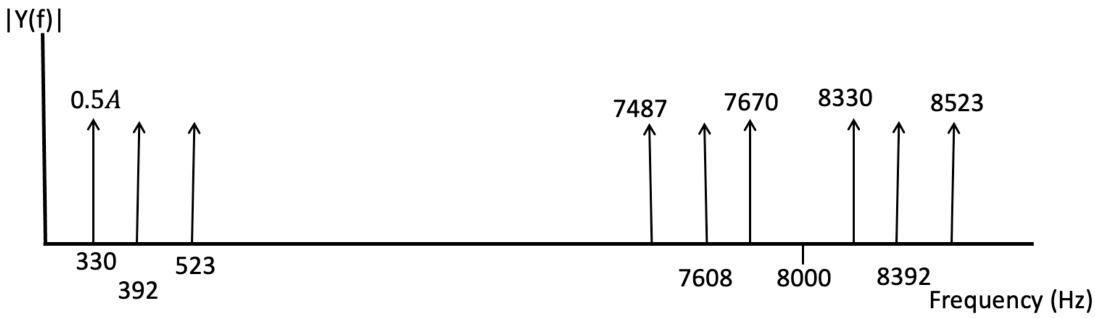
a) (When): Aliasing happens when the sampling frequency f_{samp} is less than twice the signal frequency f_{sig}.

(Why): The sampling process results in the spectrum of continuous-time signal repeated at the sampling frequency f_{samp} and integer multiple of f_{samp}. Therefore, when $f_{samp} < f_{sig}$, the signal components centred at f_{samp} extends to the baseband.

(Bad consequence): Corrupt the original signal and cannot be reversed.

(How to avoid): use a lowpass filter to limit the energy of frequency components above $f_{samp}/2$ to avoid corruption of the original signal.

b) (i)



(ii) The aliased frequency component is at $f_{samp} - 523 = 477Hz$. The chord will sound awful!

4. A torsion system with a heavy wheel W has a moment of inertia J. It is connected to a stationary anchor through a shaft S with a shaft stiffness of k as shown in Figure Q4. The movement of the wheel is damped by a friction pad F with a damping coefficient of c. An external torque T is acting on the wheel in the direction shown. The angle of rotation of the wheel α is measured from its stationary condition. The relationship between the wheel angle α and the external torque T is given by the following equation:

$$T - k\alpha - c\frac{d\alpha}{dt} - J\frac{d^2\alpha}{dt^2} = 0$$

a) Derive the transfer function $H(s)$ between the angle α and the torque T.

[6]

b) Hence or otherwise, write down the equation for the natural frequency, damping factor and the DC gain of the system in terms of J, k and c.

[12]

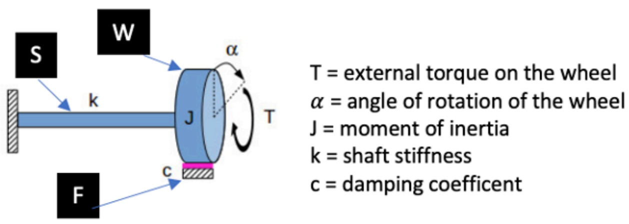


Figure Q4

This question tests students' understanding of 2nd order system, its transfer function and its dynamic behaviour.

a)

$$H(s) = \frac{\alpha(s)}{T(s)} = \frac{1}{Js^2 + cs + k} = \frac{1}{k}\left[\frac{\frac{k}{J}}{s^2 + \frac{c}{J}s + \frac{k}{J}}\right]$$

b) Compare this with the standard 2nd order system equation in Lecture 7 slide 13:

$$H(s) = \frac{b_0}{s^2 + a_1 s + a_0} = K\frac{{\omega_0}^2}{s^2 + 2\zeta\omega_0 s + {\omega_0}^2}$$

We have:

Resonant frequency $\omega_o = \sqrt{a_0} = \sqrt{\frac{k}{J}}$

Damping factor $\zeta = \frac{a_1}{2\sqrt{a_0}} = \frac{1}{2}\frac{c}{\sqrt{Jk}}$

DC gain $K = \frac{1}{k}$

5. A digital filter has an impulse response $h[n]$ as shown in Figure Q5a.

a) What is the transfer function H[z] of this filter?

[4]

b) A signal $x[n]$ shown in Figure Q5b is applied to the input of the filter. Write down the difference equation which relates the output signal $y[n]$ of the filter to its input x.

[4]

c) Using the graphical convolution method, derive the output $y[n]$ for 0 ≤ n ≤ 5.

[12]

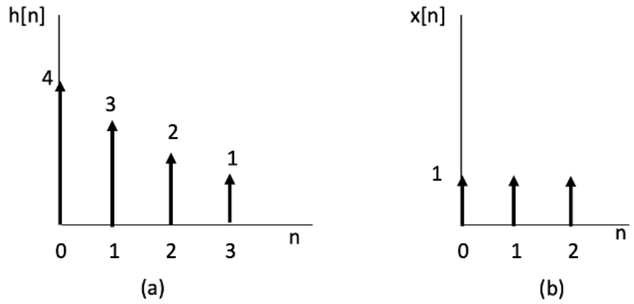


Figure Q5

This question tests students' understanding of discrete time systems, transfer function in the z-domain, difference equation and the idea of convolution.

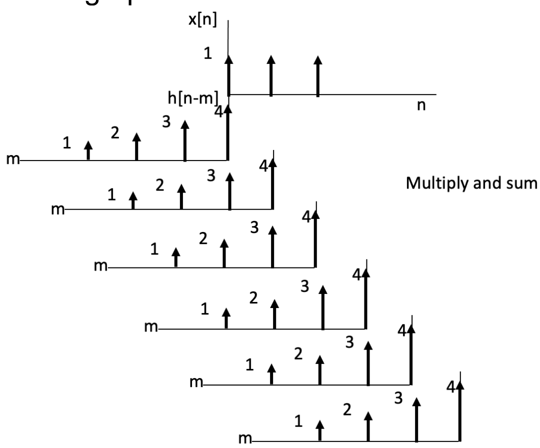
a) The transfer function is simply the z-transform of the impulse response h[n]:

$$H[z] = 4 + 3z^{-1} + 2z^{-2} + z^{-3}$$

b) The output y[n] is simply:

$$y[n] = 4x[n] + 3x[n-1] + 2x[n-2] + x[n-3]$$

c) Use graphical method:



Therefore:

Y[0] = 4, y[1] = 4+3 = 7, y]2] = 4+3+2=9, y[3]= 3+2+1=6, y[4]=2+1=3, y[5]=1.

6. Figure Q6 shows a first-order system G(s) being controlled in a feedback loop with a proportional-differential controller H(s).

   Derive the closed-loop transfer function of the system.

   [12]

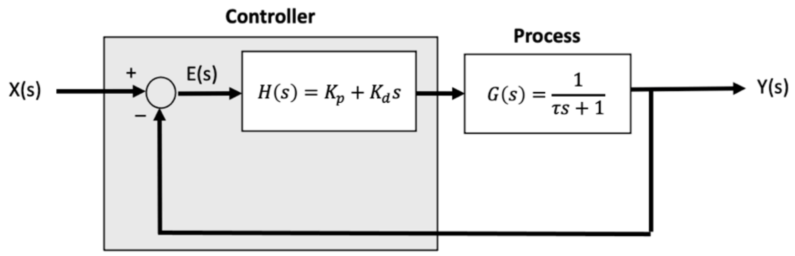


Figure Q6

The closed-loop transfer function is:

$$F(s) = \frac{Y(s)}{X(s)} = \frac{H(s)G(s)}{1 + H(s)G(s)}$$

$$= \frac{\dfrac{K_p + K_d s}{1 + \tau s}}{1 + \dfrac{K_p + K_d s}{1 + \tau s}}$$

$$= \frac{K_p + K_d s}{1 + \tau s + K_p + K_d s}$$

$$= \frac{K_p + K_d s}{(1 + K_p) + (K_d + \tau)s}$$

[END OF PAPER]